From Admissible Transition Kernels to Emergent Detailed Balance

A Collapse-Selection Reconstruction of Equilibrium Structure

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Abstract

We present a collapse-selection formulation of transition kernels over admissible relational configurations and show that detailed balance emerges as a symmetry of the induced transition structure under coarse-graining. Rather than treating equilibrium as a primitive principle, we derive it as a consequence of admissibility constraints, persistence ordering, and the suppression of directional asymmetry. This establishes equilibrium structure and effective potentials as descriptive features of collapse-stabilized sectors rather than generative drivers of dynamics.

1 Introduction

Across physics, statistical mechanics describes equilibrium structure through probability distributions and detailed balance conditions. These are typically treated as fundamental or derived from microscopic dynamical assumptions.

In this work, we take a different perspective. We begin with a collapse-selection framework in which admissibility determines which configurations persist, and introduce a transition kernel over admissible sectors. We show that under mild structural conditions, the induced transition process satisfies detailed balance, with equilibrium distributions emerging as a consequence of admissibility structure rather than underlying dynamical laws.

2 Relational Configuration Space and Kernel

Let Σ denote a relational configuration space, and let

$$\Phi : \Sigma \to \Sigma \tag{1}$$

be a collapse-selection operator.

Define the admissible sector:

$$A = \{x \in \Sigma \mid \Phi(x) = x\}. \tag{2}$$

Let $\mathcal{C}_E = \{C_\alpha\}$ denote effective configuration classes at descriptive scale E.

We define the transition kernel:

$$K(C_f, C_i; E) = \sum_{\gamma \in \mathcal{P}(C_i \to C_f)} \prod_k T_{\alpha_{k+1},\alpha_k}(E), \tag{3}$$

where γ is an admissible transition chain.

3 Transition Weight Decomposition

Each transition weight is defined as:

$$T_{\alpha\beta}(E) = A_{\alpha\beta}(E)e^{-\Gamma_{\alpha\beta}(E)}e^{i\Theta_{\alpha\beta}(E)}, \tag{4}$$

where:

- $A_{\alpha\beta}$: accessibility
- $\Gamma_{\alpha\beta}$: collapse-loss
- $\Theta_{\alpha\beta}$: relational phase

Define positive weights:

$$W_{\alpha\beta}(E) = |T_{\alpha\beta}(E)| = A_{\alpha\beta}(E)e^{-\Gamma_{\alpha\beta}(E)}. \tag{5}$$

4 Admissibility Constraints

We impose the following constraints:

1. **Admissibility exclusion:**
$$T_{\alpha\beta}(E) = 0 \quad \text{if } C_\beta \to C_\alpha \text{ is inadmissible.}$$
2. **Persistence:**
$$T_{\alpha\alpha}(E) > 0.$$
3. **Finite invariance:** Equivalent configurations yield equivalent transition weights at scale E.
4. **Nonnegativity:**
$$A_{\alpha\beta} \geq 0, \quad \Gamma_{\alpha\beta} \geq 0.$$
5. **Loss monotonicity:** Fragile transitions incur larger $\Gamma_{\alpha\beta}$.
6. **Phase composition:**
$$\Theta_{\alpha\gamma} \sim \Theta_{\alpha\beta} + \Theta_{\beta\gamma}.$$

These constraints define a family of admissible transition kernels.

5 Induced Transition Process

Define normalized transition probabilities:

$$P_{\alpha\beta}(E) = \frac{W_{\alpha\beta}(E)}{\sum_\mu W_{\mu\beta}(E)}. \tag{6}$$

This defines a stochastic process over admissible sectors.

6 Emergence of Detailed Balance

Assume:

- **(C1)** Effective closure: no net leakage outside $\mathcal{C}_E$
- **(C2)** Integrable asymmetry:
$$\frac{W_{\alpha\beta}}{W_{\beta\alpha}} = e^{-(S_\alpha - S_\beta)}$$
for some scalar function S_α
- **(C3)** Suppression of directional currents

Define:
$$\pi_\alpha = \frac{e^{-S_\alpha}}{\sum_\nu e^{-S_\nu}}. \tag{7}$$

Then:
$$\pi_\beta P_{\alpha\beta} = \pi_\alpha P_{\beta\alpha}. \tag{8}$$

Thus, detailed balance emerges.

7 Interpretation

Detailed balance is not fundamental. It arises when:

- collapse-selection defines admissible structure,
- transition asymmetry is integrable,
- directional phase effects are suppressed.

Equilibrium therefore reflects a symmetry of stabilized structure.

8 Emergent Potential

From:
$$\pi_\alpha \propto e^{-S_\alpha}, \tag{9}$$
we obtain:
$$V_\alpha \sim -\log \pi_\alpha \sim S_\alpha. \tag{10}$$

Thus, effective potentials are logarithmic compressions of admissible structure.

9 Breakdown of Detailed Balance

Detailed balance fails when:

- open systems introduce leakage,
- asymmetry is non-integrable,
- phase circulation persists,
- regime transitions occur.

10 Connection to Spectral Peaks

The preceding construction connects naturally to spectral structure. In the QCG transition-kernel framework, observable spectral peaks correspond to metastable or collapse-stable sectors of the admissible configuration space.

Let C_α denote an admissible sector. The induced equilibrium-like weight

$$\pi_\alpha \propto e^{-S_\alpha}$$

measures the relative persistence of that sector under the admissible transition kernel. A spectral peak occurs when a sector satisfies three conditions:

1. **High persistence:**
$$\pi_\alpha \gg \pi_\beta$$
for nearby sectors C_β.

2. **Low collapse leakage:**
$$\Gamma_\alpha \ll 1$$
or, more generally, Γ_α is locally minimized.

3. **Stable transition accessibility:**
$$K(C_\alpha, C_i; E)$$
is enhanced over a finite range of the descriptive parameter E.

Thus, spectral peaks are not primitive energy states. They are observable projections of sectors where the admissible transition kernel concentrates weight.

10.1 Peak Position and Peak Width

Within this interpretation:

- Peak position corresponds to the location in E-space where an admissibility basin is maximally stable.
- Peak height corresponds to the induced persistence weight π_α.
- Peak width corresponds to collapse leakage, decay, or instability.

Schematically:

$$\text{peak height} \sim \pi_\alpha,$$

$$\text{peak width} \sim \Gamma_\alpha,$$

$$\text{peak location} \sim \arg\max_E \pi_\alpha(E).$$

This reproduces the usual interpretation of resonant structure while shifting the ontology: the peak is not a fundamental object, but the projected residue of a stable admissibility basin.

10.2 Detailed Balance and Spectral Symmetry

When the admissible sector graph satisfies detailed balance,

$$\pi_\beta P_{\alpha\beta} = \pi_\alpha P_{\beta\alpha},$$

the resulting spectral structure is equilibrium-like. Peaks appear as symmetric or quasi-symmetric concentrations of transition weight around stable sectors.

When detailed balance fails, spectral structure may become asymmetric. This occurs when:

- collapse leakage persists,
- transition asymmetry is non-integrable,
- phase circulation produces directional currents,
- the system crosses between collapse regimes.

Thus, deviations from detailed balance correspond to distorted, broadened, shifted, or asymmetric spectral features.

10.3 Interpretive Consequence

The connection may be summarized as:

> Detailed balance describes when admissible transition structure becomes equilibrium-like; spectral peaks reveal where that equilibrium-like structure becomes observable.

In this sense, spectral peaks are experimental signatures of collapse-stabilized admissibility basins. The equilibrium distribution identifies which sectors persist, while spectral observation identifies where those persistent sectors appear under projection.

11 Conclusion

We have shown that detailed balance emerges naturally from admissible transition kernels under collapse-selection. Equilibrium structure is not a generative principle, but a symmetry of stabilized structure.

> Collapse defines admissibility. The kernel defines connectivity. Equilibrium emerges as symmetry of the resulting structure.